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THREE-DIMENSIONAL EDGE WAVES ON CURVED TOPOGRAPHIES

by

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ABSTRACT

Exact solutions are found to the linearized three dimensional equations for free surface gravity waves trapped against a straight coastline with a variable (perpendicular to the coastline) topography. Three families of topographies are found, one concave upwards and two convex upwards, which will support these edge waves as separable solutions to the original equations. For a given topography, specified by an initial slope, h_o', a typical water depth, h_M, and a typical offshore distance, x_M, solutions are given in terms of the nondimensional parameters, $h_o' x_M/h_M$, and h_o'. The first parameter is a measure of how the initial slope h_o' compares to that for a straight topography from the origin to (x_M, h_M), i.e. (h_M/x_M). This parameter characterizes the families in ranges $0 < h_o' x_M/h_M < 1/2$, $1/2 < h_o' x_M/h_M < 1$ and $1 < h_o' x_M/h_M < (\ln 10)/0.9$. (This latter constant bound is a function of the definition of offshore scale and can be modified to other values.) The nondimensional frequency, period, vertical wave number, offshore decay rate and topography can be expressed in terms of the single parameter $h_o' x_M/h_M$; the velocities, wave height and alongshore wave number depend on both parameters.

INTRODUCTION

The trapping of wave energy along coastlines has been of theoretical interest since the original edge wave solution of Stokes (1846). Recently, there has been consideration of the physical reality of such trapped waves and a number of observations have led to a belief that such trapped waves may indeed play a role in beach erosion, local resonances, anisostatic response to traveling pressure disturbances, etc.; a review of these points is contained in LeBlond and Mysak (1977) and (1978). Some distinction must be made at the outset of any research as to the type of trapped waves to be discussed. Here the emphasis is on class I waves, i.e. gravity dominated, of frequencies high enough that Coriolis effects may be neglected, i.e. edge waves. These waves are basically high frequency, dispersive and can travel in either direction along a coastline; in fact, since no rotational effects are included and thus no direction is preferred, the coast line may be oriented in any direction.

The problem at hand then is to examine the existence of edge waves traveling along and trapped against a straight coastline bounding a semi-infinite ocean whose depth contours are parallel to the coastline. The ocean is considered to be a perfect homogeneous fluid, inviscid and irrotational. The purpose of the paper is to establish exact solutions for these three dimensional equations by reversing the usual question "what are the trapped solutions for a given topography" to read "what topographies will support trapped wave solutions". One solution already available for comparison is that of Stokes (1846) who considered a linear topography (constant slope) and obtained a "zeroth" mode solution which Ursell (1952) extended to a number of discrete modes plus a continuous spectrum. This was based on the previous work of Eckart (1951) who obtained higher modes using a shallow water (2D) approximation.

Other authors [e.g. Reid (1958)] added Coriolis effects to the shallow water theory. In fact, the bulk of the effort in this area has relied on the shallow water assumption [e.g. see p. 460 LeBlond and Mysak (1977)] and the lack of other exact three dimensional edge wave solutions was commented upon by Grimshaw (1974) who examined upper and lower bounds on such dispersion relationships. Some non-linear topographies have been considered. Robinson (1964) examined class II, i.e. quasi-geostrophic low frequency, waves on a linear sloping shelf of finite width terminated by a discontinuous drop to a constant depth infinite ocean. Ball (1966) used an exponential depth profile with solutions in the form of hypergeometric or Jacobi polynomials to examine both edge and continental shelf waves, i.e. both class I and class II waves. Mysak (1968) also discussed both classes, using Robinson's depth profile, where the solution is given in terms of Laguerre polynomials. Hidaka (1976-a) examined seiching due to a submarine bank described by a parabolic depth variation in terms of Mathieu functions. This is a trapped but not specifically an "edge" wave, but it is of class I nevertheless. In a related paper (Hidaka (1976-b)) he examined shelf resonances when the apex of the parabola reaches the free surface. Murty (1977) gives a general review of tsunami theory and current research, which includes theoretical and observational studies on edge waves caused by tsunamis (which are generally of high enough frequency that Coriolis effects may be neglected, i.e. class I waves) but are long enough that a two dimensional shallow water theory suffices.

Experiments performed by Ursell (1952) indicated the reality of such edge waves; further experiments by Galvin (1965) indicated that these could be excited (nonlinearly) by normally incident waves. Huntley and Bowen (1973) and Huntley (1976) have made observations which indicate the presence of edge waves along coastlines in the U.K. while Nakamura (1962), Hatori and Takahasi (1964), Nakamura and Watanabe (1966), Aida (1967), Hatori (1967) and others have described the excitation of such edge waves by tsunamis incident onto the coast of Japan. There is other observational evidence, Wilson and Torum (1968), that some tsunami energy is trapped on the continental shelf in the generating region at least for the 1964 Alaskan Earthquake and it may be expected that some of this energy could be converted to and travel along the coastline as edge waves. Olsen and Hwang (1974) indicate that edge waves may play a significant role in near shore tsunami behavior.

FORMULATION

The governing equations are well known, e.g. Lamb (1932) and are written here for a straight coastline, $x = 0$, $-\infty < y < +\infty$, bounding a semi-infinite ocean, $x > 0$ with a free surface at $z = 0$ and a bottom topography, $z = -h(x)$ with z positive upward:

(1) $\nabla^2\phi(x,y,z,t) = 0; \; x > 0, \; -h < z < 0, \; -\infty < y < \infty$

(2) $\sigma^2\phi - g\,\partial\phi/\partial z = 0; \quad x > 0, \quad z = 0, \; -\infty < y < \infty$

(3) $\partial\phi/\partial z = -(dh/dx)\,\partial\phi/\partial x; \quad x > 0, \quad z = -h, \quad -\infty < y < \infty$

where a time harmonic behavior, $\exp(-i\sigma t)$ has been assumed for the velocity potential, ϕ. The Coriolis effect has been neglected and the y dependence will be taken as periodic (the edge wave assumption). Assuming separation of variables, the velocity potential is given as $\phi = X(x)\,Z(z)\cos(ky-\sigma t)$, using a real form for the y and t dependence, leading to:

(4) $X(x) = A\exp(-\ell x) + B\exp(+\ell x)$

(5a) $Z(z) = C\sinh(nz) + D\cosh(nz)$

(5b) $\quad\quad = C'\sin(mz) + D'\cos(mz)$

where ℓ^2 is a separation constant and $n^2 = k^2 - \ell^2 > 0$ and $m^2 = \ell^2 - k^2 > 0$ provide two different solution forms for $Z(z)$. The coefficient B is taken to be zero to provide a "trapped" solution and A may be absorbed into $Z(z)$.

Substituting the first solution, (5a), into the boundary conditions provides

(6) $C/D = \sigma^2/gn$

(7) $dh/dx = -(n/\ell)\dfrac{[\tanh(nh) - \sigma^2/gn]}{[1 - (\sigma^2/gn)\tanh(nh)]}$

(a) $\quad\quad = -(n/\ell)\tanh[nh - \tanh^{-1}(\sigma^2/gn)]; \quad \sigma^2/gn < 1$

(b) $\quad\quad = -(n/\ell)\coth[nh - \coth^{-1}(\sigma^2/gn)]; \quad \sigma^2/gn > 1$

Using the boundary conditions in (5b) leads to

(8) $C'/D' = \sigma^2/gm$

(9) $dh/dx = (m/\ell)\tan[mh + \tan^{-1}(\sigma^2/gm)]$

Equations (7) and (9) can be integrated to give three families of topographies that give trapped solutions as summarized below:

Case I: $k^2 > \ell^2$; $\sigma^2/gn < 1$

Case II: $k^2 > \ell^2$; $\sigma^2/gn > 1$

Case III: $k^2 < \ell^2$; all σ^2/gm

SPECIFIC SOLUTIONS

Case I leads to a concave upward topography with $h(0)$ set equal to zero (to prevent flux across the coastline), an asymptotic constant depth, $h_\infty = [\tanh^{-1}(\sigma^2/gn)]/n$ and an initial slope, $h'_o \equiv (dh/dx)_o = \sigma^2/g\ell$. Using $\theta = \tanh^{-1}(\sigma^2/gn)$,

$$h(x) = [\theta - \sinh^{-1}(\sinh\theta \cdot \exp[-n^2x/\ell])]/n$$

with a velocity potential given as

$$\phi = D\exp(-\ell x)\cos(ky-\sigma t)\cosh[n(z+h_\infty)]/\cosh(nh_\infty)$$

Case II ($k^2 > \ell^2$, $\sigma^2/gn > 1$, i.e. eq. 7b) leads to a convex upward topography with an infinite slope at a finite value of h and x. Using $\theta' = \coth^{-1}(\sigma^2/gn)$,

$$h(x) = [\theta' - \cosh^{-1}(\cosh\theta' \cdot \exp[-n^2x/\ell])]/n$$

The maximum extent of the topography is $x_M = (\ell/n^2)\ell n[\cosh\theta']$ which occurs at $h_M = \theta'/n$. The velocity potential in this case is, for $x < x_M$,

$$\phi = D\exp(-\ell x)\cos(ky-\sigma t)\sinh[n(z+h_M)]/\sinh(nh_M)$$

When $\sigma^2/gn = 1$, both cases I and II recover the Stokes solution, with $h = x\tan\alpha$, $\ell = -k\cos\alpha$, $n = -k\sin\alpha$ and $\alpha = \tan^{-1}(h'_o)$.

Case III ($k^2 < \ell^2$, all σ^2/gm; i.e. eq. 9) using $\gamma = \tan^{-1}\frac{\sigma^2}{gm}$ leads again again to a convex upward topography with an infinite slope at $x_M = -(\ell/m^2)\ell n[\sin\gamma]$ with depth $h_M = (\pi/2 - \gamma)/m$

$$h(x) = [\sin^{-1}(\sin\gamma \cdot \exp[+m^2x/\ell]) - \gamma]/m$$

with a velocity potential for $x < x_M$ given by

$$\phi = D'\exp(-\ell x)\cos(ky-\sigma t)\sin[m(h_M+z)]/\sin(mh_M)$$

These last two families of solutions both appear to be physically unrealistic in that the prescribed bottom topography does not extend past a finite distance and finite depth before "doubling back." If, however, the resulting velocity fields for these problems have decayed sufficiently with distance from the straight coastline through the term $\exp(-\ell x)$ and/or if they have decayed sufficiently with depth to have both u and w negligible at the lower boundary before these "turning points" are reached, the point is moot since the lower boundary condition, eq. (3), is essentially satisfied identically for all x and z past these turning points. Thus these solutions may provide a basis for realistic trapped waves on a topography which matches that prescribed by these solutions prior to the turning points and a flat (or any other) topography past them.

The rest of this report will examine the implications of these solutions, particularly for the various limiting cases. To do so conveniently, it is useful to nondimensionalize the parameters and variables used. The coordinates x, y and z are nondimensionalized with respect to h_M($h_\infty = h_M$ for case I) as are the parameters k, ℓ and n (m will be counted with n generically), i.e. $(\bar{x},\bar{y},\bar{z}) = (x,y,z)/h_M$ and $(\bar{k},\bar{\ell},\bar{n}) = (k,\ell,n)\,(h_M)$ as the case requires. The frequency is scaled to $\sigma_o = (g/h)^{1/2}$ as is the time, i.e. $\bar{\sigma} = \sigma/\sigma_o$ and $t' = t \cdot \sigma_o$. Velocities, $\bar{v} = \nabla\phi$, are scaled by $u_o = u(x=0,\ y=0,\ z=0,\ t=0)$ and the wave height, $\xi = -(1/g)\frac{\partial\phi}{\partial t}(z = 0)$, by $\xi_o = \xi(x=0,\ y=\pi/2k,\ z=0,\ t=0)$ which equals $(\sigma/g\ell)u_o$.

SOLUTIONS AND LIMITING CASES

The above solutions are defined by three dimensional parameters or two nondimensional parameters, using h_M as a typical length scale. The choice of those parameters to be prescribed is important since it establishes how simply the results may be described. One such nondimensional "fundamental" parameter will be chosen as the initial slope, h_o'. The second choice depends on which aspect of the problem is under study. For examination of various "asymptotic solutions," the parameter $\bar{\sigma}^2/\bar{n}$ (or θ, θ' or γ) arises naturally in the derivation and will therefore be used for these cases. The remaining computations for the general solutions however will be carried out using $h_o' \bar{x}_M$ as the second "fundamental" parameter. For case I, x_M is defined as $h(x = .9\, x_M) = .9\, h_\infty$; actually other choices could equally well be made to define a typical offshore length, e.g. $h(x = (1-\delta)x_M) = (1-\delta)h_\infty$. Then $\bar{x}_M$ is x_M/h_M and the parameter $h_o' \bar{x}_M$ is the ratio of the actual initial slope to the slope of a straight line from the origin to (x_M, h_M). This parameter characterizes the three families as well as did $\bar{\sigma}^2/\bar{n}$.

Case I requires $1 < h_o' \bar{x}_M < (\ln 10)/.9$ (or more generally, $(\ln 1/\delta)/(1-\delta)$) which is a curve whose initial slope h_o' is greater than h_M/x_M giving a curve lying below the straight line from the origin to $(.9\, x_M, .9\, h_M)$ for $\bar{x} < x_M$, i.e. is concave upwards. Case II and case III require $0.5 < h_o' \bar{x}_M < 1$ and $0 < h_o' \bar{x}_M < 0.5$ respectively; these both give topographies lying above (closer to the surface) that the straight line to (x_M, h_M), i.e. are concave downwards.

Thus the three families can be represented by a continuous range of values of $h_o' \bar{x}_M$, with some question as to the cross over values of 1.0 (which corresponds to the Stokes solution) and 0.5 (which presents no actual difficulty). The main advantage to use of this parameter is that the frequency,

$\bar{\sigma}$, the vertical "wavenumber," $\bar{n}$, the (modified) offshore decay rate, $\bar{l}/\bar{x}_M$, and the topography, $\bar{h}(\bar{x}/\bar{x}_M)$ can be represented as functions of $h_o' \bar{x}_M$ alone; i.e. they are one parameter functions. The along shore wavenumber, $\bar{k}$, the wave height, $\bar{\xi}$, and the velocities, $\bar{u}$, $\bar{v}$, $\bar{w}$, depend on both $h_o' \bar{x}_M$ and h_o', i.e. are two parameter functions. Thus much of the solution can be expressed quite simply in terms of a single parameter.

CASE I:

This is in many ways the most satisfying solution with a well behaved topography. Consider h_∞, h_o' and $x_M = (1/.9)x(h = .9h_\infty)$ to be specified. In non-dimensional terms, $\bar{n}$ is given by h_o' and $\bar{x}_M$ since $\bar{h}_M$ is 1 by definition:

$$\frac{\tanh \bar{n}}{\bar{n}} \cdot \ln\left[\frac{\sinh \bar{n}}{\sinh \cdot 9\bar{n}}\right] = .9\, h_o' \bar{x}_M$$

with $\bar{\sigma}$, $\bar{\ell}$ and $\bar{k}$ given by

$$\bar{\sigma} = [\bar{n} \tanh \bar{n}]^{1/2}$$

$$\bar{\ell} = [\bar{n} \tanh \bar{n}]/h_o'$$

$$\bar{k} = \bar{n}[1 + (\tanh^2\bar{n})/h_o'^2]^{1/2}$$

If $\bar{\sigma}^2/\bar{n}$ were specified instead of $h_o' \bar{x}_M$, $\bar{n}$ would be simply $\tanh^{-1}(\bar{\sigma}^2/\bar{n})$, eliminating the need to solve a transcendental equation for $\bar{n}$. The topography is given by

$$\bar{h}(\bar{x}) = 1 - (1/\bar{n})\sinh^{-1}[\sinh \bar{n} \cdot \exp(-\bar{n}^2\bar{x}/\bar{\ell})]$$

and the velocities and wave height by

$$\bar{u} = \exp(-\bar{\ell}\bar{x})\cos(\bar{k}\bar{y} - \bar{\sigma}\bar{t})\cosh[\bar{n}(1+\bar{z})]/\cosh \bar{n}$$

$$\bar{v} = (\bar{k}/\bar{\ell})\exp(-\bar{\ell}\bar{x})\sin(\bar{k}\bar{y} - \bar{\sigma}\bar{t})\cosh[\bar{n}(1 + \bar{z})]/\cosh \bar{n}$$

$$\bar{w} = -(\bar{n}/\bar{\ell})\exp(-\bar{\ell}\bar{x})\cos(\bar{k}\bar{y} - \bar{\sigma}\bar{t})\sinh[\bar{n}(1 + \bar{z})]/\cosh \bar{n}$$

$$\bar{\zeta} = \exp(-\bar{\ell}\bar{x})\sin(\bar{k}\bar{y} - \bar{\sigma}\bar{t})$$

Note that, as $\bar{n}$ approaches zero, $h_o' \bar{x}_M$ approaches 1 while as $\bar{n}$ approaches infinity, $h_o' \bar{x}_M$ approaches $(\ln 10)/.9$. Furthermore, $\bar{n}$ and $\bar{\sigma}$ depend only on the combination $h_o' \bar{x}_M$, while $\bar{\ell}$, $\bar{k}$, $\bar{h}$ and the velocities require separate values for h_o' and $\bar{x}_M$. However, $\bar{\ell}/\bar{x}_M$ and $\bar{h}(\bar{x}/\bar{x}_M)$ depend only on $h_o' \bar{x}_M$.

A: Consider the limiting case $\bar{\sigma}^2/\bar{n} = \varepsilon << 1$. Approximations for the governing parameters are found to be

$$\bar{n} \approx \varepsilon$$
$$\bar{\ell} \approx \varepsilon^2/h_o'$$
$$\bar{\sigma} \approx \varepsilon$$
$$\bar{k} \approx \varepsilon[1 + (\varepsilon/h_o')^2]^{1/2}$$
$$h_o' \, \bar{x}_M \approx (1/.9)[\ell n\ 10 - \varepsilon^2((\ell n\ 10)/3 - 1/\ )]$$

The restriction on $\bar{\sigma}^2/\bar{n}$ implies $\sigma << \sigma_o$, i.e. that a shallow water wave theory be used leading to the same topography for all applicable frequencies. The topography is given by

$$\bar{h}(\bar{x}) = 1 - \exp[-h_o' \, \bar{x}]$$

which in turn is the same as that used by Ball (1967). Within the limitations of this case, the velocity potential is

$$\phi = D \exp[-\kappa^2\bar{x}/h_o']\cos[\kappa(1 + (\kappa/h_o'^2)^{1/2}\bar{y} - \bar{\sigma}\bar{t}]$$

with $\kappa \equiv \bar{\sigma}$. This corresponds to Ball's solution for the case of zero rotation and mode number $(n) = 0$, i.e. the first term in his hypergeometric polynomial which then reduces to a simple exponential decay in $\bar{x}$. The corresponding velocity field has no vertical variation, an exponential decay with distance perpendicular to the shore line and a periodic wave behavior along the shore line, both of which depend on the nondimensional frequency and the initial beach slope. Since this form has already been obtained by Ball, in far greater generality, it will not be discussed further here.

B: Consider next the case when $\bar{\sigma}^2/\bar{n}$ is near 1, i.e. $\bar{\sigma}^2/\bar{n} \approx 1-\varepsilon$; $\varepsilon \ll 1$. Then $\bar{n} \simeq \frac{1}{2}\ln(2/\varepsilon)$ is a large number. The remaining parameters follow as

$$\bar{\sigma} \approx [\tfrac{1}{2}\ln(2/\varepsilon)]^{1/2}$$

$$\bar{\ell} \approx (1/2\ h_o')\ln(2/\varepsilon)$$

$$\bar{k} \sim (1/2)\ln(2/\varepsilon)[1 + (1/h_o')^2]^{1/2}$$

and

$$h_o'\ \bar{x}_M \approx 1 - \ln(1 - (\varepsilon/2)^{.1})/(.45\ \ln(2/\varepsilon))$$

i.e. $h_o'\ \bar{x}_M$ is slightly larger than 1.

The topography is given by

$$\bar{h}(\bar{x})\quad 1 - (1/2\ln(2/\varepsilon))^{-1}\sinh^{-1}\left[(\varepsilon/2)^{(h_o'\bar{x}-1)/2}\right]$$

For $h_o'\bar{x} \ll 1$, this reduces to a linear topography, $\bar{h} = h_o'\bar{x}$. Since $h_o'\ \bar{x}_M$ is close to 1, this implies that $\bar{x} \ll \bar{x}_M$ for this linear topography. For these small values of $\bar{x}$, the velocities and wave height are

$$\bar{u} \approx (2/\varepsilon)^{1/2(\bar{z}-\bar{x}/h_o')}\cos[\bar{y}\cdot(1/2\ln(2/\varepsilon)(1+(1/h_o')^2)^{1/2} - (1/2\ln(2/\varepsilon))^{1/2}\ \bar{t}]$$

$$\bar{v} \approx (1 + h_o'^2)(2/\varepsilon)^{1/2(\bar{z}-\bar{x}/h_o')}\ \sin[\quad\text{"}\quad\quad\text{"}\quad]$$

$$\bar{w} \approx -h_o'(2/\varepsilon)^{1/2(\bar{z}-\bar{x}/h_o')}\ \cos[\quad\text{"}\quad\quad\text{"}\quad]$$

$$\bar{\xi} \sim (2/\varepsilon)^{-\bar{x}/2h_o'}\ \sin\quad[\quad\text{"}\quad\quad\text{"}\quad]$$

This solution recovers the classical Stokes' solution, but only for large $\bar{n}$ and $\bar{\sigma}$. However, if the general solution is examined, the complete Stokes' solution (for all $\bar{\sigma}$) is obtained for $\bar{\sigma}^2/\bar{n}$ exactly equal to 1.0. This limiting process for case I can only provide the high frequency Stokes' solution since the regular Stokes' solution is actually neither in class I nor in class II but on their boundary. Since $\bar{z} \leq 0$, these velocities die out rapidly with increasing depth and distance offshore.

CASE II:

This, together with case III, compose the solutions for $h'_o \bar{x}_M < 1$, i.e. the concave downward topographies; $\bar{n}$ is related to $h'_o \bar{x}_M$ through

$$h'_o \bar{x}_M = \frac{\coth \bar{n}}{\bar{n}} \cdot \ln(\cosh \bar{n})$$

with

$$\bar{\sigma} = (\bar{n} \coth \bar{n})^{1/2}$$

$$\bar{\ell} = (\bar{n} \coth \bar{n})/h'_o$$

$$\bar{k} = \bar{n}[1 + (\coth^2\bar{n})/h'^2_o]^{1/2}$$

and

$$\bar{h}(\bar{x}) = 1 - (1/\bar{n})\cosh^{-1}[\cosh \bar{n} \cdot \exp(-\bar{n}^2\bar{x}/\bar{\ell})]$$

The velocities are then given by

$$\bar{u} = \exp(-\bar{\ell}\bar{x})\cos(\bar{k}\bar{y} - \bar{\sigma}\bar{t})\sinh[\bar{n}(1 + \bar{z})]/\sinh \bar{n}$$

$$\bar{v} = (\bar{k}/\bar{\ell})\exp(-\bar{\ell}\bar{x})\sin(\bar{k}\bar{y} - \bar{\sigma}\bar{t})\sinh[\bar{n}(1 + \bar{z})]/\sinh \bar{n}$$

$$\bar{w} = -(\bar{n}/\bar{\ell})\exp(-\bar{\ell}\bar{x})\cos(\bar{k}\bar{y} - \bar{\sigma}\bar{t})\cosh[\bar{n}(1 + \bar{z})]/\sinh \bar{n}$$

and the wave height by

$$\bar{\zeta} = \exp(-\bar{\ell}\bar{x})\sin(\bar{k}\bar{y} - \bar{\sigma}\bar{t})$$

As $\bar{n}$ approaches zero, $h'_o\bar{x}_M$ approaches 1/2 while as $\bar{n}$ approaches infinity, $h'_o \bar{x}_M$ approaches 1 from below. Again, $\bar{\sigma}$, $\bar{\ell}/\bar{x}_M$, $\bar{h}(\bar{x}/\bar{x}_M)$ and $\bar{n}$ depend only on the single parameter $h'_o \bar{x}_M$.

Limiting values of $\bar{\sigma}^2/\bar{n}$ may again be examined.

A: Consider the limiting case $\bar{\sigma}^2/\bar{n} = 1 + \varepsilon$ where $\varepsilon << 1$. Here, $\bar{n} \approx 1/2\ln(2/\varepsilon)$ again and $h'_o \bar{x}_M \approx 1 - \dfrac{2 \ln 2}{\ln(2/\varepsilon)}$ with

$$\bar{\sigma} \approx [1/2\ln(2/\varepsilon)]^{1/2}$$

$$\bar{\ell} \approx (\ln(2/\varepsilon))/2h_o'$$

$$\bar{k} \approx (1/2h_o')\ln(2/\varepsilon)(1 + h_o'^2)^{1/2}$$

and, for $\bar{x}h_o' < 1$ (as it must be for $\bar{x} < \bar{x}_M$)

$$\bar{h}(\bar{x}) \approx h_o' \bar{x}$$

with

$$\bar{u} \approx (2/\varepsilon)^{(\bar{z} - \bar{x}/h_o')/2} \cos[\bar{y}\cdot(1/2 \ln(2/\varepsilon)\cdot(1 + (1/h_o')^2)^{1/2}) - (1/2 \ln(2/\varepsilon))^{1/2} \bar{t}]$$

$$\bar{v} \sim (1 + h_o'^2)^{1/2}(2/\varepsilon)^{(\bar{z} - \bar{x}/h_o')/2} \sin[\quad '' \quad '' \quad]$$

$$\bar{w} \approx -h_o'\cdot(2/\varepsilon)^{(\bar{z} - \bar{x}/h_o')/2} \cos[\quad '' \quad '' \quad]$$

$$\bar{\xi} \approx (2/\varepsilon)^{-\bar{x}/2h_o'} \sin[\quad '' \quad '' \quad]$$

For small $\bar{x}$, $\bar{z}$, the Stokes solution for large $\bar{\sigma}$ is found again as would be expected. In fact, this solution is identical to I-B except for $h_o' \bar{x}_M$ which is greater than 1 for I-B and less than 1 here for non-zero ε.

B: Next consider the limiting case $\bar{\sigma}^2/\bar{n} = 1/\varepsilon$ where $\varepsilon \ll 1$. Here, $\bar{n}$ is approximately $\varepsilon + (1/3)\varepsilon^3$, $h_o' \bar{x}_M \quad 1/2 + \varepsilon^2/12$ and

$$\bar{\sigma} \approx 1 + \varepsilon^2/6$$

$$\bar{\ell} \quad (1/h_o')(1 + \varepsilon^2/3)$$

$$\bar{k} \quad [\varepsilon^2 + (1/h_o')^2]^{1/2}(1 + \varepsilon^2/3)$$

and

$$\bar{h}(\bar{x}) \quad 1 - (1 - 2h_o' \bar{x})^{1/2}$$

which for small $h_o' \bar{x}$ is linear again with slope h_o'. The velocity field and wave height are given by

$$\bar{u} \approx (1 + \bar{z})\exp(-\bar{x}/h_o')\cos[\bar{y}\cdot(\varepsilon^2 + (1/h_o')^2)^{1/2} - \bar{t}]$$

$$\bar{v} \approx (1 + (\varepsilon\, h_o')^2]^{1/2}(1 + \bar{z})\exp(-\bar{x}/h_o')\sin[\qquad " \quad]$$

$$\bar{w} \quad - h_o'\cdot\exp(-\bar{x}/h_o')\cos[\qquad " \quad]$$

$$\bar{\xi} \quad \exp(-\bar{x}/h_o')\sin[\qquad " \quad]$$

Note that, to the lowest order in ε, $h_o' \bar{x}_M$, $\bar{\sigma}$ and $\bar{\ell}$ are essentially independent of ε, as are $\bar{k}$, $\bar{u}$, $\bar{v}$ and $\bar{w}$ if $h_o' \ll 1$ as well.
The topography is parabolic, with a maximum horizontal extent of $1/2\, h_o'$ as expected. The horizontal velocities decrease linearly with depth, but do not vanish at the bottom until $\bar{h}$ equals $\bar{h}_M$, i.e. $\bar{z}$ can reach -1.
The vertical velocity is constant with depth and is $O(h_o')$ compared to the horizontal velocities; this is then a small value unless $\bar{x}_M$ is itself of order $O(1)$, i.e. the initial slope is 45° or greater which is physically unlikely.

CASE III: $(\bar{k}^2 < \bar{\ell}^2)$

Here the range of $h_o' \bar{x}_M$ is from 0, as $\bar{m}$ approaches $\pi/2$ to 1/2, as $\bar{m}$ approaches zero with the general relation

$$-\frac{\cot \bar{m} \cdot \ell n(\cos \bar{m})}{\bar{m}} = h_o' \bar{x}_M$$

with

$$\bar{\sigma} = (\bar{m} \cot \bar{m})^{1/2}$$

$$\bar{\ell} = (\bar{m} \cot \bar{m})/h_o'$$

$$\bar{k} = \bar{m}\left[\left(\frac{\cot \bar{m}}{h_o'}\right)^2 - 1\right]^{1/2}$$

and

$$\bar{h}(\bar{x}) = \frac{\sin^{-1}[\sin \gamma \cdot \exp(+\bar{m}^2\bar{x}/\bar{\ell})] - \gamma}{\bar{m}}$$

with

$$\gamma = -\bar{m} + \pi/2 \quad \text{and}$$

$$\bar{u} = \exp(-\bar{\ell}\bar{x}) \cdot \cos(\bar{k}\bar{y} - \bar{\sigma}\bar{t}) \cdot \sin[\bar{m}(1 + \bar{z})]/\sin \bar{m}$$

$$\bar{v} = (\bar{k}/\bar{\ell})\exp(-\bar{\ell}\bar{x})\sin(\bar{k}\bar{y} - \bar{\sigma}\bar{t})\sin[\bar{m}(1 + \bar{z})]/\sin \bar{m}$$

$$\bar{w} = -(\bar{m}/\bar{\ell})\exp(-\bar{\ell}\bar{x})\cos(\bar{k}\bar{y} - \bar{\sigma}\bar{t})\cos[\bar{m}(1 + \bar{z})]/\sin \bar{m}$$

$$\bar{\xi} = \exp(-\bar{\ell}\bar{x})\sin(\bar{k}\bar{y} - \bar{\sigma}\bar{t})$$

A: The first limiting case here is $\bar{\sigma}^2/\bar{m} = \varepsilon << 1$. Then $\gamma \approx \varepsilon$ and $\bar{m} \approx \pi/2 - \varepsilon$ with $h_o' \bar{x}_M \simeq -\dfrac{2\varepsilon \ \ell n \ \varepsilon}{\pi}$

$$\bar{\sigma} \approx (\pi \ \varepsilon/2)^{1/2}$$

$$\bar{\ell} \approx \varepsilon \ \pi/2 \ h_o'$$

$$\bar{k} \approx (\pi/2)[(\varepsilon/h_o')^2 - 1]^{1/2}$$

and

$$\bar{h}(\bar{x}) \approx 2/\pi[\sin^{-1}(\exp[-\pi(\bar{x}_M - \bar{x})h_o'/2\varepsilon]) - \varepsilon]$$

Clearly $\varepsilon > h_o'$ in order to have real $\bar{k}$, i.e. a progressive wave solution. If $\bar{x}$ is small enough such that $\exp(\pi\bar{x}h_o'/2\varepsilon)$ $1 + \pi\bar{x}h_o'/2\varepsilon$ the linear topography solution is found again, i.e.

$$\bar{h} \approx h_o' \bar{x}$$

The velocities and wave height are

$$\bar{u} \approx \exp(-\varepsilon\pi\bar{x}/2h_o')\cdot\cos[\bar{y}\cdot\pi/2\cdot((\varepsilon/h_o')^2 - 1)^{1/2} - (\pi\varepsilon/2)^{1/2}\,\bar{t}]\sin[\pi/2(1 + \bar{z})]$$
$$\bar{v} \approx [1 - (h_o'/\varepsilon)^2]^{1/2}\cdot\exp(-\varepsilon\pi\bar{x}/2h_o')\sin[\quad " \quad]\sin[\pi/2(1 + \bar{z})]$$
$$\bar{w} \approx -(h_o'/\varepsilon)\exp(-\varepsilon\pi\bar{x}/2h_o')\cos[\quad " \quad]\cos[\pi/2(1 + \bar{z})]$$
$$\bar{\xi} \approx \exp(-\varepsilon\pi\bar{x}/2h_o')\sin[\quad " \quad]$$

Such a solution requires extremely small values of the initial slope, h_o'. This in turn leads to a rapid decay of wave heights and velocities in the offshore direction due to the $\exp[-\varepsilon\pi\bar{x}/2h_o']$, e.g. for ε equal to h_o' (such that k is zero corresponding to no alongshore variation), the velocities have decayed to 4% of thier shoreline values when x has reached $2\ h_M$. The wave periods are very long, which could cause Coriolis effects to become significant and thus invalidate this solution.

B: The final limiting case, $\bar{\sigma}^2/\bar{m} = 1/\varepsilon$, $\varepsilon << 1$ implies $\gamma \approx \pi/2-\varepsilon$, $\bar{m} \approx \varepsilon$ and $h_o'\ \bar{x}_M \approx 1/2$ with $\bar{\sigma} \approx 1$, $\bar{\ell} \approx (1/h_o')$, $\bar{k} \approx (1-(\varepsilon h_o')^2)^{1/2}/h_o'$ and $\bar{h}(\bar{x}) \approx 1 - (1 - 2\bar{x}h_o')^{1/2}$ which reduces to the linear form when $\bar{x}\ h_o'$ is small. The velocities and wave height in this case are

$$\bar{u} \approx (1 + \bar{z})\exp(-\bar{x}/h_o')\cos[\bar{y}(1 - (\varepsilon h_o')^2)^{1/2}/h_o' - \bar{t}]$$

$$\bar{v} \approx (1 - (\varepsilon h_o')^2)^{1/2}(1 + \bar{z})\exp(-\bar{x}/h_o')\sin[\quad " \quad]$$

$$\bar{w} \approx -h_o' \cdot \exp(-\bar{x}/h_o')\cos[\quad " \quad]$$

$$\bar{\zeta} \approx \exp(-\bar{x}/h_o')\sin[\quad " \quad]$$

which match the limiting case II-B as they must since both of these correspond to the case $h_o' \bar{x}_M = 1/2$ which is common to both case II and case III. Apart from the alongshore velocity, $\bar{v}$, and the corresponding alongshore dependence on $\bar{k}\bar{y}$, this solution is independent of ε. Furthermore, as ε passes through zero, the match between case II-B and case III-B is complete, i.e. the transition from case II to case III is smooth.

DISCUSSION OF GENERAL SOLUTION

Although the problem is completely specified by three dimensional (e.g h_o', x_M and h_M) or two nondimensional (e.g. h_o' and $\bar{x}_M$), it is of practical interest to express results in terms of as few parameters as possible. This may be done for a portion of the present problem. The quantities $\bar{n}$, $\bar{\sigma}$, $\bar{\ell}/\bar{x}_M$ are functions only of the single parameter $(h_o' \bar{x}_M)$ and the topography $\bar{h}$ is a function of the parameter $h_o' \bar{x}_M$ and the spatial variable $\bar{x}/\bar{x}_M$. Thus although the computer program, given as appendix A, uses three dimensional inputs, h_o', h_M and x_M, the results may be stated in a fairly compact form.

The vertical "wavenumber," $\bar{n}$, is related to the single parameter $h_o' \bar{x}_M$ by $h_o' \bar{x}_M = F(\bar{n})$ where $F(\bar{n})$ has different forms for the three cases considered and is plotted in Figure (1). Thus $\bar{n}$ may be read from this figure for any value of $h_o' \bar{x}_M$ between 0 and $(\ln 10)/0.9$ (or calculated through the program). Figure (2) provides the (nondimensional) period, $\bar{T}$, as a function of the single parameter, $h_o' \bar{x}_M$. Since the nondimensionalization involves h_M, dimensional periods require knowledge of both h_M and $h_o' \bar{x}_M$. At $h_o' \bar{x}_M$ equal to 1, there is an anomaly in the original solution. At this value, equation (7) reduces to dh/dx equal to a constant $(-n/\ell)$ and $h_o' \bar{x}_M$ is 1 for all values of h_o'. In a sense, this is a "fourth" solution family, the well known Stokes solution, in that it can not be obtained from the other solutions, except as a high frequency limit. Thus Figure (2) has zero period at $h_o' \bar{x}_M$ equal to 1 with the Stokes solution indicated as a dashed line. The Stokes solution, $h = + \tan\alpha \cdot x$, requires $\sigma^2 = gk \sin\alpha$. Thus for any $h_o' = \tan\alpha$, there is a single specific frequency, σ, at least for the lowest separable mode. Ursell's extension (1952) to higher modes is not comparable here since those higher modes were not separable solutions and could not be obtained by this present approach.

A question then arises as to the appropriate solution for an "almost" linear topography. The examples here, which represent the only separable solutions--but clearly not necessarily the only solutions--will only support very high frequency trapped waves when $h_o' \bar{x}_M$ is near to 1 but not equal to 1.

The offshore decay parameter, $\bar{\ell}$, depends on both $h_o' \bar{x}_M$ and $\bar{x}_M$ but in a manner such that $\bar{\ell}/\bar{x}_M$ is a function of $h_o' \bar{x}_M$ only. This is shown in Figure (3) where $\bar{\ell}/\bar{x}_M$ is given for h_M = 200 meters and x_M = 200 kilometers for various values of h_o' but valid of course for any choices of h_M and x_M. This figure indicates the limited range of practical solutions since all terms decay like $\exp(-\bar{\ell}\bar{x})$ or equivalently $\exp(-(\bar{\ell}/\bar{x}_M)(\bar{x}/\bar{x}_M)\bar{x}_M^2)$. To have this exponential greater than 0.04 compared to a value of 1 at $\bar{x} = 0$, $(\bar{\ell}/\bar{x}_M) < \pi/(\bar{x}/\bar{x}_M)(\bar{x}_M)^2$. Even for $\bar{x}/\bar{x}_M$ up to 0.1 representing the region of measurable wave heights, $(\bar{\ell}/\bar{x}_M) < 31.4/\bar{x}_M^2$. Realistic choices for $\bar{x}_M$ are of the order of 40 to 1000 for the continental slope and shelf respectively; these limit $\bar{\ell}/\bar{x}_M < 2 \times 10^{-2}$ and 3×10^{-5} which in turn imply that $h_o' \bar{x}_M$ must be close to either zero or $(\ell n\ 10)/0.9$ in either case. Thus for realistic ocean topographic scales, the energy must be trapped very near shore.

Figure (4) shows the topography, h, as a function of $\bar{x}/\bar{x}_M$ for several values of the parameter $h_o' \bar{x}_M$, based on a choice of h_M equal to 200 meters and x_M equal to 200 kilometers but again valid in general. Those topographies for $h_o' \bar{x}_M > 1$ are forced to pass through (0.9, 0.9) by the choice of definition for x_M; other choices would simply rescale these curves to pass through $(1-\delta, 1-\delta)$, i.e. appear steeper for $\delta < 0.1$ or shallower for $\delta > 0.1$. As $h_o' \bar{x}_M$ approaches 1.0, the topography appears almost linear except near $\bar{x}/\bar{x}_M$ equal to 1. For values of $h_o' \bar{x}_M$ less than 1, convex upward topographies are found with an infinite vertical slope at $\bar{x}/\bar{x}_M$ equal to 1.0. The transition across $h_o' \bar{x}_M$ equal to 0.5 is smooth with no ascertainable peculiarities.

Calculations of velocities and waveheights produced extremely small values for any offshore distance that itself was not a very small fraction of x_M. For example, with $h_o' = 0.002556$, $h_M = 200$ meters, $x_M = 200$ kilometers the velocity and waveheight values had reached 4% of their maximum values when x reached 0.2% of x_M, e.g. about 400 meters--at which distance the water depth was approximately one meter. This implies that the edge wave was completely trapped in the region where the model is the poorest representation of the overall topography and is therefore inadequate. For this reason, no results are shown for this case.

For much smaller values of $\bar{x}_M$ however, it is possible to illustrate the depth dependence for velocities. Figures (5) and (6) show results for $\bar{x}_M = 1.0$. The first plots the offshore velocity, $\bar{u}$, as a function of $(\bar{x}/\bar{x}_M)$ and $\bar{z}$ for $h_o' = 1.01$. The second illustrates the different coastline values for $\bar{v}$ and $\bar{w}$ as compared to the reference value of 1 for $\bar{u}$, as a function of $h_o' \bar{x}_M$. There is a cutoff at $h_o' = 0.2854$ for $\bar{x}_M = 1.0$ below which k becomes imaginary. This corresponds to the requirement that $\bar{m} > \ln(\sec \bar{m})$. Similarly near $h_o' \bar{x}_M = (\ln 10)/0.9$, $\bar{v}$ behaves like $(1 + (h_o'/\epsilon)^2)^{1/2}$, as defined in case I-A, while $\bar{w}$ behaves like $-h_o'$.

CONCLUSION

The bulk of observations have been for shallow water-long wavelength situations such that the question of vertical variation does not enter, as befits the general emphasis on two dimensional theory for edge waves. All tsunami observations are of this form as were the observations of Huntley and Bowen (1973) who found evidence for the lowest mode edge wave of Ball's (1967) solution corresponding to case I-A here. Ursell (1952) described experiments on a linear topography but did not give any details of vertical structure. Thus it appears that separate observations and/or experiments may be necessary to detect the three dimensionality of those edge waves described here. Whether such observations are warranted is another question. Wavelengths that are significant enough to carry appreciable energy alongshore may also need to be long enough to allow a two dimensional theory to apply. Such a theory allows for far more general topographies than does the three dimensional theory, as discussed in appendix B. This is clearly a question for further consideration on physical grounds.

Assuming this research to be physically justified, there are still several mathematical questions left unanswered. First, are there non-separable solutions, analogous to Ursell's (1952) solutions, for these or related topographies and if so how may they be found? Second, what is the appropriate three dimensional edge wave solution for an "almost" linear topography? Is it close to the original Stokes solution or must it be close to the high frequency limit given here?

One extension of this work would be towards alongshore variations in topography. If the topography varied periodically along the coastline (i.e. in y) the edge waves would be filtered as Floquet waves, e.g. Brillouin (1953), and only certain frequencies and wavelengths could propagate. Alternatively, the effect of a small irregularity in the otherwise straight coastline could be studied, e.g. as by Fuller and Mysak (1977).

The extension to lower frequencies such that Coriolis effects become significant does not appear to be appropriate in context of a three dimensional theory since these lower frequencies would imply shallow water-long wave theory to be a good approximation in the real ocean. This then brings up the question of the applicability of two dimensional theories to the present examples. Case I-A is essentially shallow water theory but the remaining asymptotic solutions do have some depth variation as does the general solution. Thus two dimensional theories give quite different results than those given here as seen in appendix B. Even case III-A, which is a low frequency solution, requires very small h_o' such that a two dimensional solution may apply for $\bar{x} << \bar{x}_M$ eventually (for $\bar{x}$ near $\bar{x}_M$) vertical variation is required.

However over the regions of interest, i.e. where the waves have not decayed to negligible values, even the asymptotic solutions do not exhibit appreciable vertical variations for realistic values of h_o' and $\bar{x}_M$ since $\bar{z}$ never got appreciably different from 1 (the free surface). There are clearly values for the input parameters where these vertical variations may be significant, but they do not appear to be physically realistic, e.g. $\bar{x}_M < 1$.

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APPENDIX A

The following computer program accepts a single card IJOB as the number of jobs (input data sets) and then several "jobs" each with h_o', h_M and x_M (in cm) as input. The card IVEL = 0 behind IVEL = 1 will surpass velocity output while the reverse will provide velocities at $\bar{x}/\bar{x}_M$ = 0.0 (0.1) 1.0 and $\bar{z}/\bar{h}$ = 0.0 (0.2) 1.0. The remaining output is labeled; the symbol D in front of a variable means the dimensional form (in cgs units). The symbol HPX is $h_o' \bar{x}_M$, HOP is h_o', SIGMAO is σ_o, etc.. Typical running times for twenty different input cards is about 2.5 seconds on a CDC CYBER 173.

```
      PROGRAM TAXELL (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION H1(101),H2(101),H3(101)
      DIMENSION U(41,41),V(41,41),W(41,41)
      DIMENSION F1(21) ,F2(21),F3(21)
      REAL K1,M1,K2,M2,K3,M,L1,L2,L3
      ATANH(X) = 0.5*ALOG( (1.+X)/(1.-X))
      ACOTH(X) = 0.5*ALOG( (1.+X)/(X-1.))
      ACOSH(X) = ALOG( X + SQRT(X**2 - 1.))
      ASINH(X) = ALOG( X + SQRT(X**2 + 1.))
      SINH(X) = 0.5*( EXP(X) - EXP(-X))
      COSH(X) = 0.5*( EXP(X) + EXP(-X))
      TANH(X) = SINH(X)/COSH(X)
      COTH(X) = 1./TANH(X)
      FF1(X) = TANH(X)*ALOG( SINH(X)/SINH(X/10.)) /(X*0.9)
      FF2(X) = ALOG(COSH(X))/(X*TANH(X))
      FF3(X) =-ALOG( COS(X))/(X*TAN(X))
      G = 980.
      PI = 3.1415926
       IVAL =
      IVAL =
      ERRX = 0. 001
      ERR1 =  .01
      ERRN = 0.001
      ICOUNT = 0
      READ(5,3) IJOB
    3 FORMAT(I4)
    1 CONTINUE
    2 CONTINUE
      READ(5,1 ) HOP,HM,XM
    1 FORMAT(3F10.0)
      XM = XM . 01
      ...
      PRINT*,4H HOP= ,HOP,5H HM= ,HM,5H XM= ,XM
      SIGMAO = SQRT(G/HM)
      PRINT*,8H SIGMAO= ,SIGMAO
C     ALL VARIABLES ARE NONDIMENSIONALIZED FROM HERE ON
      HPX = HOP/HM
      PRINT*,5H HPX= ,HPX
      IF(HPX .LT. 1.0) GO TO 270
      XM1 = XM/HM
   20 PRINT*,    CASE 1 HPX .GT. 1.0
      F1(1) = ALOG(10.)  / .9
      DO 51 J = 2,11
      X = J- 1.
      F1(J) = FF1(X)
C     PRINT*, 4H Y= ,Y,8H F1(J)= ,F1(J)
   51 CONTINUE
      IF(F1(11) - HPX) 66,67,68
   66 DO 71 J = 1,10
      TEST = (F1(J)-HPX)*(F1(J+1) - HPX)
      IF( TEST .LE. 0.0) GO TO 72
   71 CONTINUE
      PRINT*,4 H CANT FIND ROOT TO F1 IN RANGE
      GO TO 2
   72 XL = J  - 1.
      XR = J
      DO 73 I = 1,21
```

```
      XM = (XR + XL)/2.
      FXM1 = FF1(XM)
      IF( XL .EQ. 0.0) FXL1 = F1(1)
      IF( XL .GT. 0.0) FXL1 = FF1(XL)
C     PRINT*, 5H XM= ,XM, 10H FF1(XM)=  ,FXM1 ,10H FF1(XL)= ,FXL1
      TL = ( FXM1 - HPX)*( FXL1 - HPX)
      IF(TL .LT. 0.0) XR = XM
      IF(TL.GE. 0.0) XL = XM
      IF ( ABS(XR-XL) .LE. ERRX) GO TO 74
   73 CONTINUE
      PRINT*,40H F1 HALVING ROUTINE DID NOT PRODUCE ROOT
      PRINT*,5H XR= ,XR,5H XL= ,XL
      GO TO 55
   74 N1 = (XR + XL)/2.
      GO TO 64
   67 N1 = 10.
      GO TO 64
   68 CONTINUE
      PRINT*,55H THIS VALUE OF HPX CORRESPONDS TO N1.GT. 1
      N1 = 10.
      DO 76 J = 1,21
      OLDN1 = N1
      N1 = -F.0*ALOG( 1.0-EXP(-0.P*.1*(HPX - 1.0)))
      IF( ABS( (N1- OLDN1)/N1) .LT. ERRN1) GO TO 64
   76 CONTINUE
      PRINT*,45H N1 ITERATION .GT. 1  DOES NOT CONVERGE
      PRINT*,5H N1= ,N1,5H DN1=  ,OLDN1
      GO TO 55
   64 CONTINUE
      PRINT*, 6H N1 = ,N1
      IF(N1 .GT. 49.)  S1 = SQRT (N1)
      IF(N1 .LE. 49.)
     1S1 = SQRT( N1*TANH(N1))
      L1 = S1**2/HOP
      K1 = SQRT(N1**2 + L1**2)
      PRINT*,5H N1= ,N1,5H L1= ,L1,5H K1= ,K1,5H S1= ,S1,6H XM1=
      DN1 = N1/HM
      DL1 = L1/HM
      DK1 = K1/HM
      DS1 = SIGMA0*S1
      PRINT*,6H DN1= ,DN1,6H DL1= ,DL1,6H DK1= ,DK1,6H DS1= ,DS1
      T1 = 2.*PI/S1
      DT1= 2.*PI/DS1
      PRINT*,6H DT1= ,DT1 ,5H T1= ,T1
      DO 15 I = 1,11
      X = (I-1)*XM1/10.
      ARG1 = SINH(N1) * EXP(-N1**2*X/L1)
      H1(I) = 1. - ASINH(ARG1)/N1
      Y = X/XM1
      PRINT*, 4H X= ,Y,5H H1= ,H1(I)
      IF(IVEL.EQ. 0) GO TO 28
      DO 155 IZ = 1,6
      DZ = H1(I)/ 5.
      Z = -(IZ-1)*DZ
      XND = X/XM1
      XL1 = X*L1
      IF(XL1.GT. 100.) FAC = 0.
```

```
      IF(XL1.LE. 1 . ) FAC = EXP(-XL1)/COSH(N1)
      U(I,IZ) = FAC*COSH(N1*(1.+Z))
      V(I,IZ) = (X1/L1)*FAC*COSH(N1*(1.+Z))
      W(I,IZ) = -(N1/L1)*FAC*SINH(N1*(1.+Z))
      WRITE(6,95) Y,XND,Z,U(I,IZ),V(I,IZ),W(I,IZ)
   95 FORMAT(6E15.5)
  125 CONTINUE
   25 CONTINUE
   15 CONTINUE
      GO TO 55
  270 IF( HPX.LT. 0.5) GO TO 29
   27 PRINT*, 40H CASE 2 HPX .LT. 1.0  AND .GT.  0.5
      X = ) HM
      F2(1) = .F
      DO 81 J = 2,11
      Y = J-1.
      F2(J) = FF2(Y)
C     PRINT*,4 X= ,Y,9H F2(J)= ,F2(J)
   81 CONTINUE
      IF(F2(11) - HPX) 86,87,88
   88 DO 91 J = 1,10
      TEST = (F2(J) - HPX)*( F2(J+1) - HPX)
      IF(TEST .LT. 0. ) GO TO 92
      IF( F2(J) .EQ. HPX ) N2 = J - 1
      IF( F2(J) .EQ. HPX ) GO TO 84
   91 CONTINUE
      PRINT*,4 H CANT FIND ROOT TO F2 IN RANGE
      GO TO 55
   92 XL = J-1.
      XR = J
      DO 93 I = 1,21
      XM = (XR + XL)/2.
      FXM2 = FF2(XM)
      IF(XL .EQ. 0.0) FXL2 = F2(1)
      IF( YL .GT.  .0) FXL2 = FF2(XL)
C     PRINT*, XM= ,XM,10H FF2(XM)= ,FXM2,10H FF2(XL)= ,FXL2
      TL = (FXM2 - HPX)*(FXL2 - HPX)
      IF(TL .LT. 0.0) XR = XM
      IF( TL .GE. 0.0) XL = XM
      IF(ABS(XR - XL) .LE. EPRX ) GO TO 94
   93 CONTINUE
      PRINT*,4 H F2 HALVING ROUTINE DID NOT PRODUCE ROOT
      PRINT*, XR= ,XR,5H XL= ,XL
      GO TO 55
   94 N2 = (XR + XL)/2.
      GO TO 84
   87 N2 = 1.
      GO TO 84
   86 CONTINUE
      PRINT*,  H THIS VALUE OF HPX CORRESPONDS TO N2.GT. 10
      N2 = ALOG(2.)/(1. - HPX)
      DO 121 I = 1,21
      OLDN2 = N2
      IF( N2 .LE. 49. ) EFAC = EXP(- 2.*N2)
      IF( N2 .GT. 49.) EFAC = 0.
      N2 = ( -ALOG(2.) + EFAC) / ( HPX/(1. + 2.*EFAC) - 1. )
      IF( ABS(N2 - OLDN2)           .LT. ERRN) GO TO 121
```

```
      120 CONTINUE
          PRINT*,40H N2 ROOT .GT. 10 DOES NOT CONVERGE
          PRINT*,5H N2= ,N2,8H OLDN2= ,OLDN2
          GO TO 84
      121 PRINT*,5H N2= ,N2
       84 CONTINUE
          PRINT*,5H N2= ,N2
          IF(N2 .EQ. 0.0) GO TO 55
          IF( N2 .GT. 49.) S2 = SQRT(N2)
          IF(N2 .LE. 49.)
         1S2 = SQRT(N2/TANH(N2) )
          L2 = S2**2/N2
          K2 = SQRT( N2**2 + L2**2)
          PRINT*,5H N2= ,N2,5H L2= ,L2,5H K2= ,K2,5H S2= ,S2,5H XM2= ,
          DS2 = S2*SIGMA0
          DN2 = N2/HM
          DL2 = L2/HM
          DK2 = K2/HM
          PRINT*,6H DN2= ,DN2,6H DL2= ,DL2,6H DK2= ,DK2,6H DS2= ,DS2
          T2 = 2.*PI/S2
          DT2 = 2.*PI/DS2
          PRINT*,6H DT2 = ,DT2  ,5H T2= ,T2
          DO 25 I = 1,11
          X = (I-1)*XM2/10.
          IF(I.EQ.11) X = 0.999*XM2
          ARG2 = COSH(N2)*EXP(-N2**2*X/L2)
          H2(I) = 1. - ACOSH(ARG2)/N2
          Y = X/XM2
          PRINT*,4H Y= ,Y,5H H2= ,H2(I)
          IF(IVEL.EQ.0) GO TO 25
          DO 255 IZ = 1,6
          DZ = H2(I) / 5.
          Z = -(IZ-1)*DZ
          XN2 = X/XM2
          XL2 = X*L2
          IF(XL2.GT. 100.) FAC = 0.
          IF(XL2.LE. 100.)    FAC = EXP(-XL2)/SINH(N2)
          U(I,IZ) = FAC*SINH(N2*(1.+Z))
          V(I,IZ) = (K2/L2)*FAC*SINH(N2*(1.+Z))
          W(I,IZ) = -(N2/L2)*FAC*COSH(N2*(1.+Z))
          WRITE(6,95) X,XN2,Z,U(I,IZ),V(I,IZ),W(I,IZ)
      255 CONTINUE
       25 CONTINUE
       29 CONTINUE
          IF( HPX .GT. 0.5) GO TO 55
          PRINT*, 50H CASE 3 HPX .LT. 0.5
          XM3 = XM/HM
          F3(1) = 0.5
          DO 101 J = 2,11
          X = (J-1)*PI/20.
          F3(J) = FF3(X)
    C     PRINT*,4H X= ,X,8H F3(J)= ,F3(J)
      101 CONTINUE
          DO 111 J = 1,10
          TEST = (F3(J) -HPX)*( F3(J+1) - HPX)
          IF(TEST .LT. 0.0) GO TO 112
          IF( F3(J) .EQ. HPX ) X = J-1
```

```
      IF( F3(J) .GE. HPX ) GO TO 56
  111 CONTINUE
      PRINT*,6H CAN'T FIND ROOT TO F3 IN THIS RANGE
      GO TO 66
  112 XL = (J-1)*PI/20.
      XR = J*PI/20.
      DO 113 I = 1,21
      XM = (XR + XL)/2.
      FXM3 = FF3(XM)
      IF(XL .EQ. 0.0) FXL3 = F3(1)
      IF( XL .GT. 0.0) FXL3 = FF3(XL)
C     PRINT*,4H XM= ,XM, 10H FF3(XM)=  ,FXM3, 10H FF3(XL)= ,FXL3
      TL = (FXM3 - HPX)*(FXL3 - HPX)
      IF(TL .LT. 0.0) XR = XM
      IF( TL .GE. 0.0) XL = XM
      IF( ABS(XR - XL) .LE. ERRX) GO TO 114
  113 CONTINUE
      PRINT*,40H F3 HALVING ROUTINE DID NOT PRODUCE ROOT
      PRINT*,4H XR= ,XR,5H XL= ,XL
      GO TO 55
  114 M = (XR + XL)/2.
   56 CONTINUE
      PRINT*, 3H M= ,M
      IF ( M.LE. 0.0) GO TO 55
      S3 = SQRT(M/TAN(M) )
      L3 = S3**2/HOP
      IF(L3.LT.M) GO TO 45
      K3 = SQRT(L3**2 - M**2)
      PRINT*,4H M= ,M,5H L3= ,L3,5H K3= ,K3,5H S3= ,S3,6H XM3= ,
      GAMMA = PI/2. - M
      DM = M/HM
      DL3 = L3/HM
      DK3 = K3/HM
      DS3 = S3*SIGMAC
      PRINT*,4H DM = ,DM,6H DL3= ,DL3,6H DK3= ,DK3,6H DS3= ,DS3
      T3 = 2.*PI/S3
      DT3 = 2.*PI/DS3
      PRINT*,6H DT3= ,DT3 ,5H T3= ,T3
      DO 30 I = 1,11
      X = (I-1)*XM3/10.
      IF(I.EQ.11) X = 0.999*XM3
      HZ(I) = (ASIN( SIN(GAMMA)*EXP(M**2*X/L3)) - GAMMA) /M
      Y = X/XM3
      PRINT*,4H Y= ,Y,5H HZ= ,HZ(I)
      IF(IVEL.EQ.0) GO TO 35
      DO 755 IZ = 1,6
      DZ = HZ(I)/5.
      Z = -(IZ-1)*DZ
      XMD = X/XM3
      XL3 = X*L3
      IF(XL3.GT. 100.) FAC = 0.
      IF(XL3.LE. 100.)  FAC = EXP(-XL3)/SIN(M)
      U(I,IZ) = FAC*SIN(M*(1.+Z))
      V(I,IZ) = (K3/L3)*FAC*SIN(M*(1.+Z))
      W(I,IZ) = -(M/L3)*FAC*COS(M*(1.+Z))
      WRITE(6,35) X,XMD,Z,U(I,IZ),V(I,IZ),W(I,IZ)
  755 CONTINUE
```

```
          35 CONTINUE
             GO TO 55
          45 PRINT*,L3,M,40H L3.LT.M
          55 CONTINUE
290          ICOUNT = ICOUNT + 1
             IF(ICOUNT .LT. IJOB) GO TO 1
             END
```

SYMBOLIC REFERENCE MAP (R=1)

Y POINTS
.1 MAXWELL

ABLES	SN	TYPE	RELOCATION				
3 ARG1		REAL		3757	ARG2	REAL	
27 DK1		REAL		3754	DK2	REAL	
70 DK3		REAL		3726	DL1	REAL	
53 DL2		REAL		3767	DL3	REAL	
56 DM		REAL		3725	DN1	REAL	
52 DN2		REAL		3731	DS1	REAL	
51 DS2		REAL		3771	DS3	REAL	
32 DT1		REAL		3755	DT2	REAL	
73 DT3		REAL		3736	DZ	REAL	
47 EFAC		REAL		3704	ERRN	REAL	
00 ERRN1		REAL		3677	ERRY	REAL	
42 FAC		REAL		3721	FXL1	REAL	
45 FXL2		REAL		3767	FXL3	REAL	
20 FXM1		REAL		3744	FXM2	REAL	
62 FXM3		REAL		14337	F1	REAL	ARRAY
64 F2		REAL	ARRAY	10411	F3	REAL	ARRAY
74 G		REAL		3765	GAMMA	REAL	
05 HM		REAL		3714	HMP	REAL	
10 HPX		REAL		3775	H1	REAL	ARRAY
42 H2		REAL	ARRAY	4377	H3	REAL	ARRAY
717 I		INTEGER		3702	ICOUNT	INTEGER	
703 IJOB		INTEGER		3676	IVEL	INTEGER	
735 IZ		INTEGER		3712	J	INTEGER	
663 K1		REAL		3665	K2	REAL	
667 K3		REAL		3671	L1	REAL	
672 L2		REAL		3673	L3	REAL	
670 M		REAL		3664	N1	REAL	
666 N2		REAL		3723	OLDN1	REAL	
746 OLDN2		REAL		3675	PI	REAL	
707 SIGMAO		REAL		3724	S1	REAL	
759 S2		REAL		3764	S3	REAL	
714 STEST		REAL		3722	T1	REAL	
731 T1		REAL		3755	T2	REAL	
772 T3		REAL		4454	U	REAL	ARRAY
675 V		REAL	ARRAY	13116	W	REAL	ARRAY
713 X		REAL		3715	XL	REAL	
741 XL1		REAL		3765	XL2	REAL	
774 XL3		REAL		3716	XM	REAL	
711 XM1		REAL		3743	XM2	REAL	

APPENDIX B: TWO DIMENSIONAL (SHALLOW WATER) EDGE WAVES

Consider the conditions under which a long wavelength free surface gravity wave may be trapped against a straight coastline by a topography which varies in a direction normal to the coastline. Taking x in this normal direction and y along the coastline, the linearized shallow water-long wavelength equations for a homogeneous fluid on a rotating earth are [e.g. LeBlond and Mysak (1978)]:

$$\partial u/\partial t + g\ \partial\eta/\partial x = fv$$

$$(1)\qquad \partial v/\partial t + g\ \partial\eta/\partial y = -fu$$

$$\partial\eta/\partial t + H(\partial u/\partial x + \partial v/\partial y) + u\partial H/\partial x = 0$$

where (u,v) are velocities, η is the free surface elevation, H is the water depth and f is the Coriolis parameter.

Assume solutions which propagate along the coastline, e.g.

$$u(x,y,t) = U(x)\cos(ky - \sigma t)$$

$$v(x,y,t) = V(x)\sin(ky - \sigma t)$$

$$\eta(x,y,t) = E(x)\sin(ky - \sigma t)$$

The original equations then reduce to

$$gE' + \sigma U - fV = 0$$

$$(2)\qquad -\sigma V + gkE + fU = 0$$

$$-\sigma E + H(U' + kV) + uH' = 0$$

Clearly U and V may be eliminated, leaving a single equation on E, e.g. LeBlond and Mysak (1978 - p. 220)

$$(3) \qquad (HE')' + \left\{\frac{\sigma^2-f^2}{g} - k^2H - \frac{fk}{\sigma} H'\right\}E = 0$$

This equation does not force a particular x dependence on either E or H, as was the case for the three dimensional equations, since here E and H are coupled while previously they were uncoupled. Instead there are many solutions. If for example, E is required to be exponentially decaying such as $E_o \exp(-\ell x)$, equation (3) requires $H(x)$ to be

$$H(x) = \frac{\sigma^2-f^2}{g(k^2-\ell^2)} + H_o \exp\left(-\left(\frac{k^2-\ell^2}{\ell+fk/\sigma}\right)x\right)$$

where H_o is an arbitrary constant defined in general either by choosing $H(0) = 0$ or $U(0) = 0$ to have no net mass flux across the coastline, $x = 0$. If $H(0)$ is chosen to be zero, the topography is given by

$$H(x) = \frac{\sigma^2-f^2}{g(k^2-\ell^2)} \left\{1 - \exp\left(-\left(\frac{k^2-\ell^2}{\ell+fk/\sigma}\right)x\right)\right\}$$

Certain ranges of parameters give negative topographies which must be excluded. Requiring that $\ell > 0$, but recognizing that k may be either positive or negative yields the following results:

For $\sigma > f$, $k > \ell$ gives a positive concave upwards topography, exponentially approaching a uniform depth, H_∞ equal to $(\sigma^2-f^2)/g(k^2-\ell^2)$; $-\ell < k < \ell$ gives a positive, convex upwards, exponentially increasing depth, $-\sigma\ell/f < k < -\ell$ gives a positive, concave upwards topography again exponentially asymptotic to H_∞ and finally $k < -\sigma\ell/f$ yields a negative topography.

For $\sigma < f, k > \ell$ and $-\sigma\ell/f < k < \ell$ both yield negative topographies, but $-\ell < k < -\sigma\ell/f$ yields a positive, concave upwards topography again decaying exponentially to the asymptotic depth H_∞ while $k < -\ell$ yields a positive, convex upwards exponentially increasing depth.

The choice of $U(0)$ equal to zero requires either $\sigma = -fk/\ell$, leading to a constant depth, $H = f^2/g\ell^2$, or the trivial solution, $E(0)$ equal to zero and thus $E(x)$ equal to zero everywhere. This choice is then rejected for this particular form for $E(x)$.

In general, equation (3) may be considered to define E for a given H or alternatively H for a given E. While the first view is more realistic physically, the second is simpler mathematically, i.e.

$$\left\{E' - \frac{fk}{\sigma} E\right\}H' + \left\{E'' - k^2E\right\} H = \left\{\frac{f^2-\sigma^2}{g}\right\} E$$

may be solved completely for $H(x)$ for a given E.

$$H(x) = C_o \exp\left[- \int \frac{E''(x)-k^2E(x)}{E'(x)- \frac{fk}{\sigma} E(x)} dx\right]$$

$$+ \left(\frac{f^2-\sigma^2}{g}\right) \exp\left[- \int \frac{E''(x)-k^2E(x)}{E'(x)- \frac{fk}{\sigma} E(x)} dx\right]$$

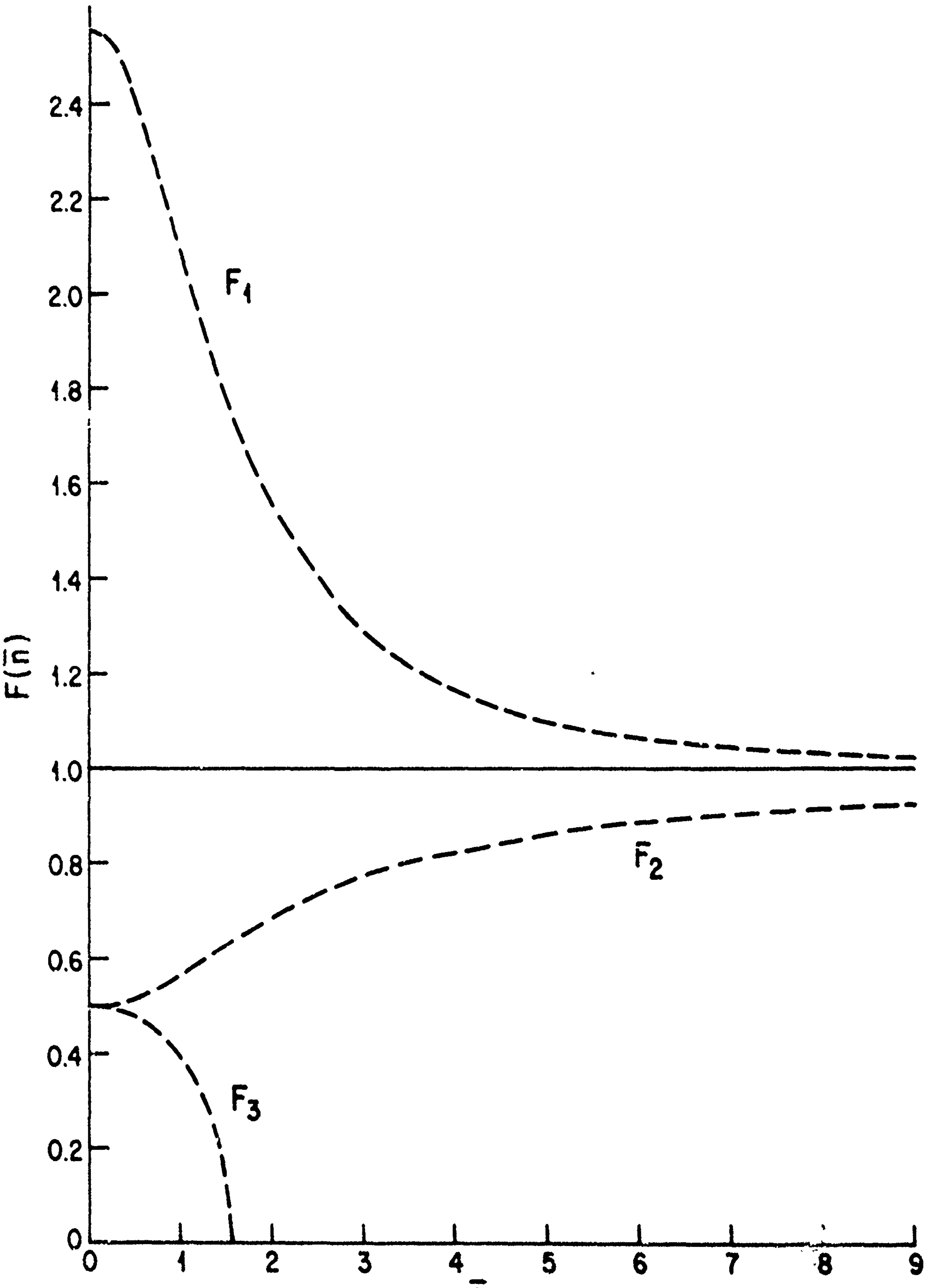
$$* \int \frac{E(x)}{E'(x)- \frac{fk}{\sigma} E(x)} \cdot \exp\left[+ \int \frac{E''(x)-k^2E(x)}{E'(x)- \frac{fk}{\sigma} E(x)} dx\right]dx$$

where C_o is an arbitrary constant to be chosen to have zero net mass flux across $x = 0$.

Although this format may seem inappropriate on physical grounds, it represents a "transformation" of $E(x)$ into $H(x)$, i.e. trial functions for E lead directly to a functional form for H and in a sense to the two dimensional counterpart to the three dimensional case studied in the main text. The alternative view of specifying H and solving for E leads to a second order differential equation on E with variable coefficients some of which vanish when H is zero, i.e. represent singular solution points. A number of solutions in this format have been generated, e.g. Ball (1967) obtained a solution in terms of hypergeometric polynomials for an exponential depth variation of the form $H_o(1-\exp(-ax))$, Hidaka (1976-b) obtained a solution in terms of modified Mathieu functions for a parabolic depth variation of the form $H_o(1 + (x^2/a^2))^{1/2}$, Robinson (1964) used a finite width, linear shelf terminating at a discontinuity to a constant depth ocean with solutions in terms of Laguerre functions, Eckart (1951), Mysak (1968) have obtained solutions in terms of Kummer functions or Laguerre polynomials for a linear topography, $H(x) = \gamma x$, and Shaw (1977) has obtained solutions in terms of (both) Kummer functions for the case of a linear topography which did not necessarily break the surface, $H(x) = H_o + \gamma x$. Other solutions could be generated for other topographies using the standard functions of mathematical physics, e.g. Bateman (1954).

In a sense, however, these solutions are all the same type; series solutions, using the method of Frobenius, to the original differential equations with the singular solution (if any) at the origin surpressed, if necessary.

LIST OF FIGURES



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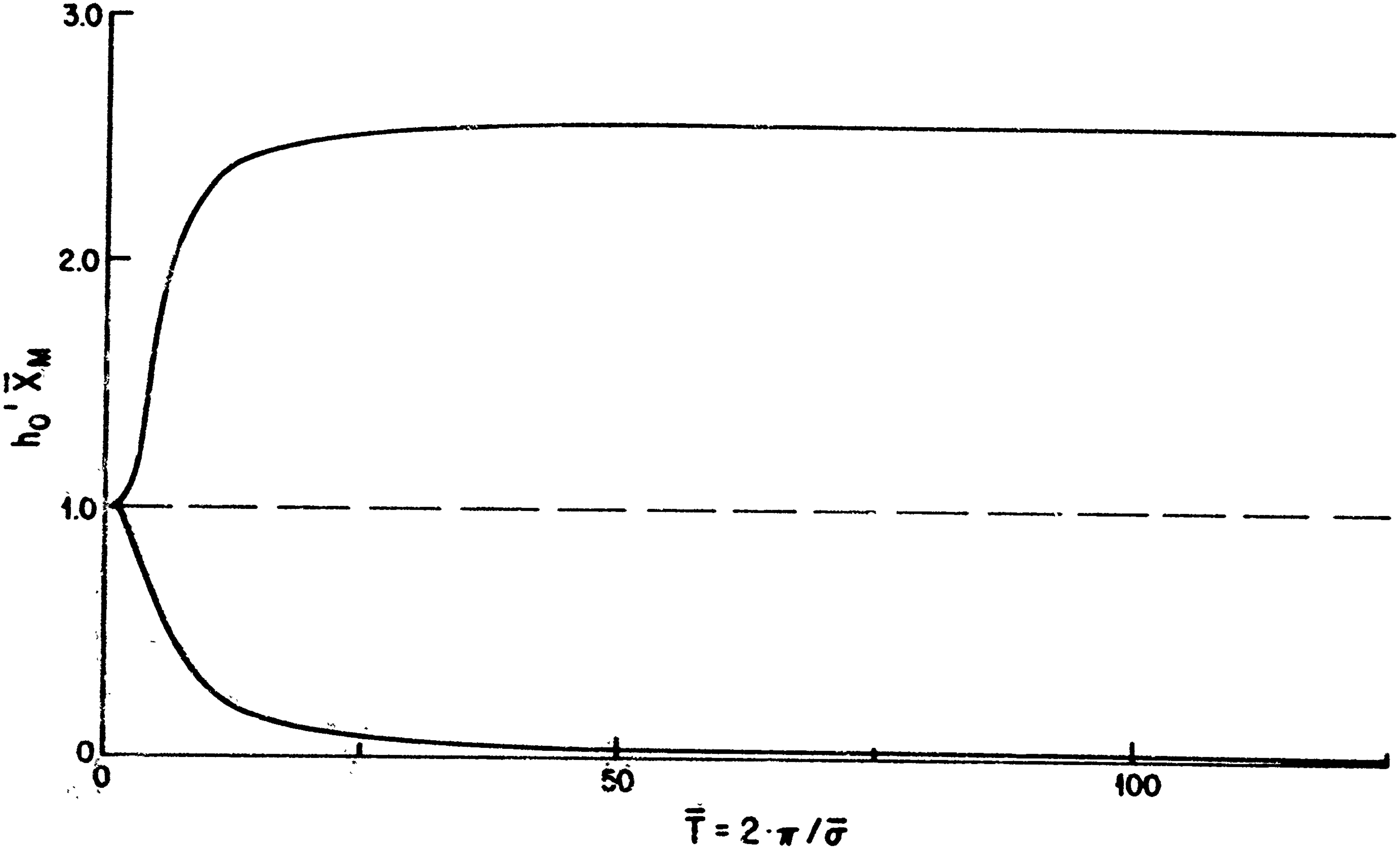

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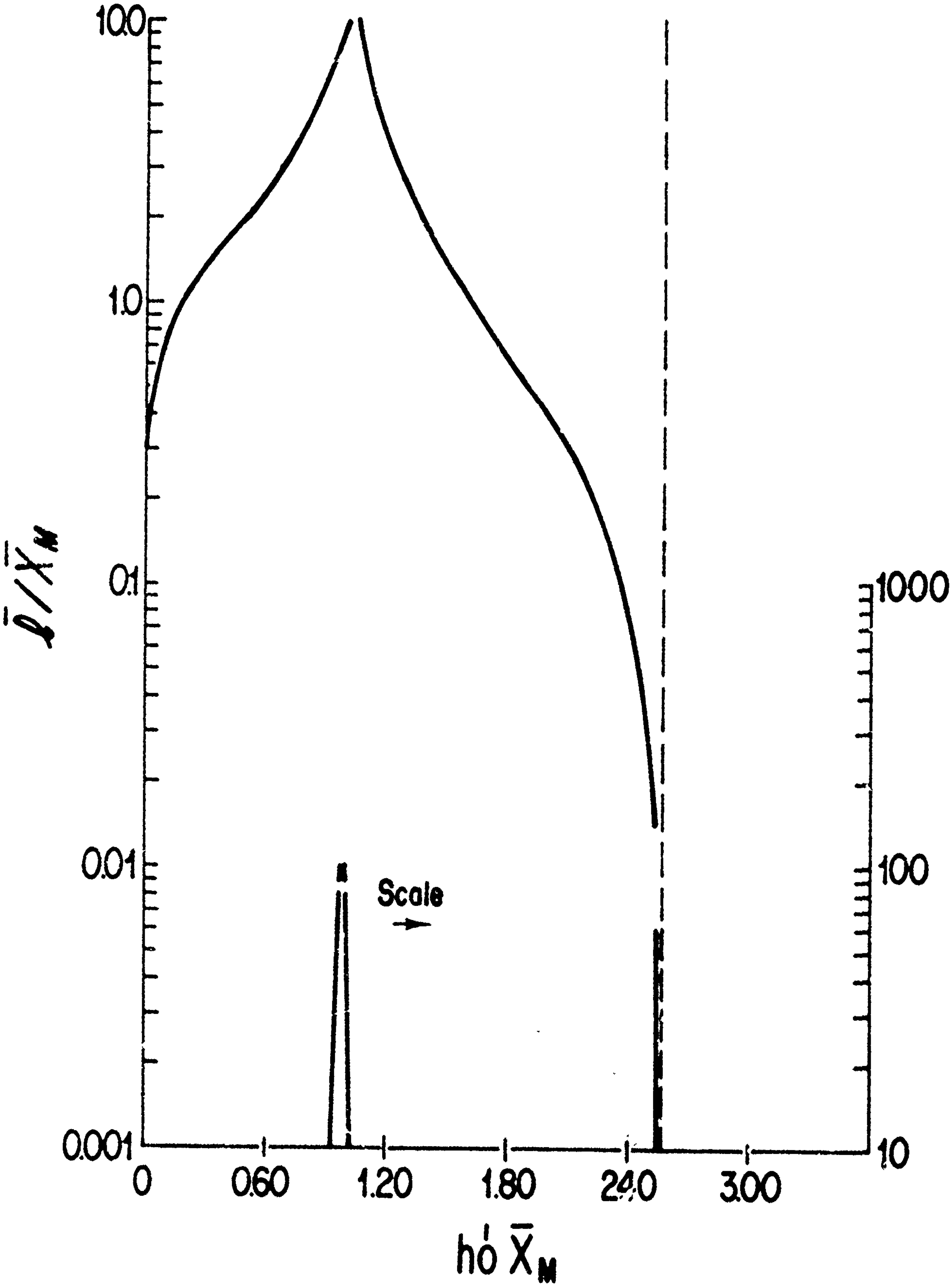
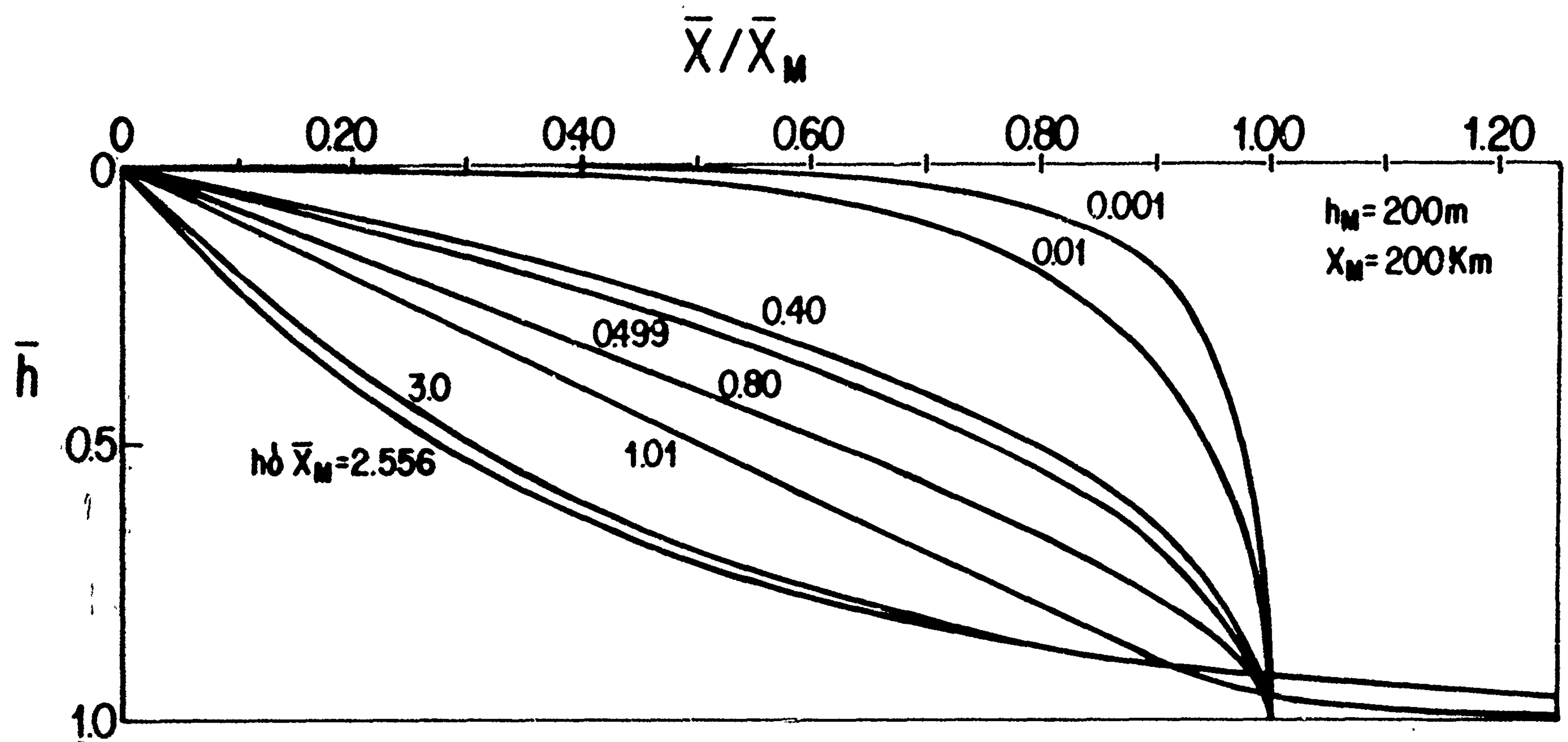
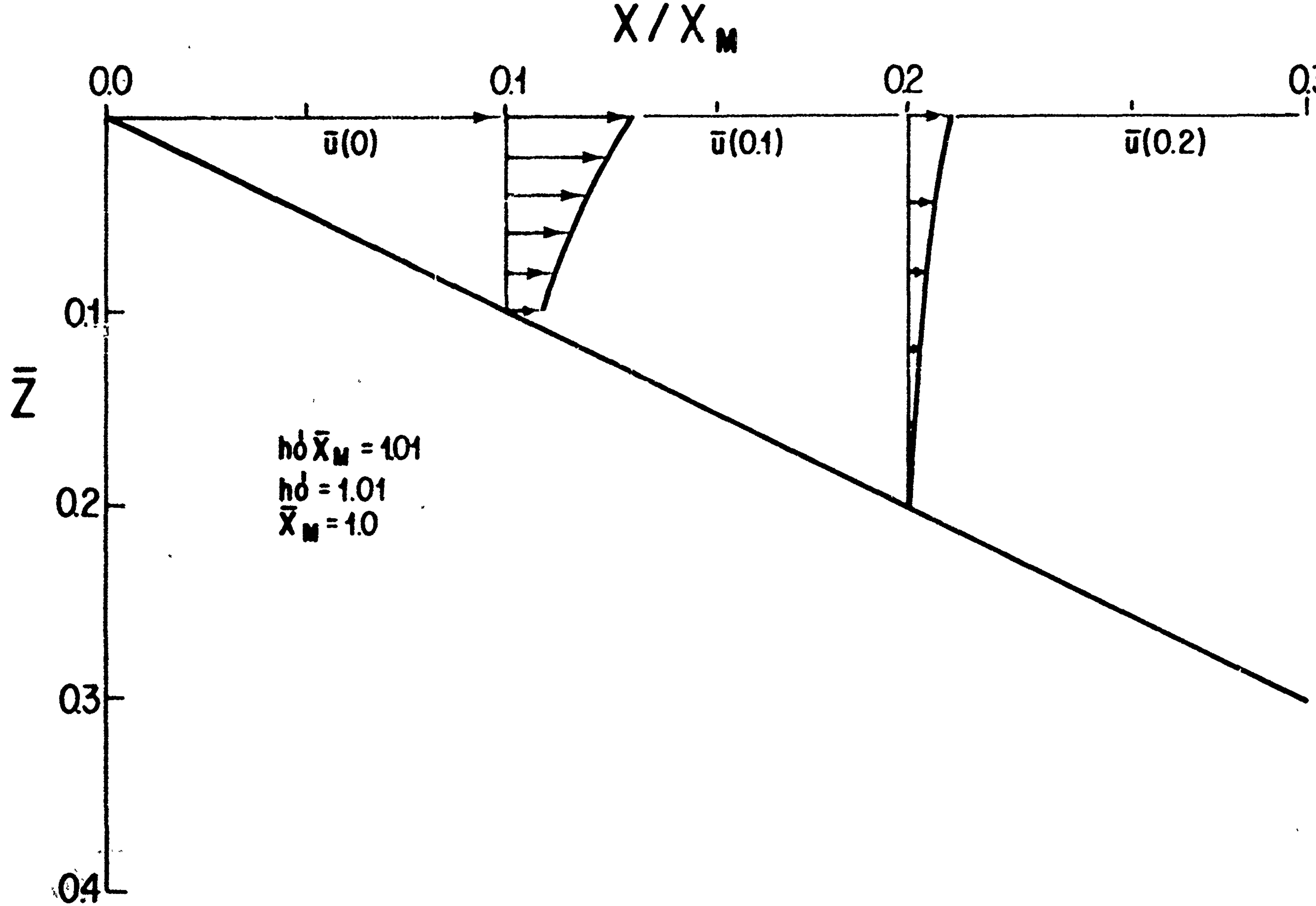


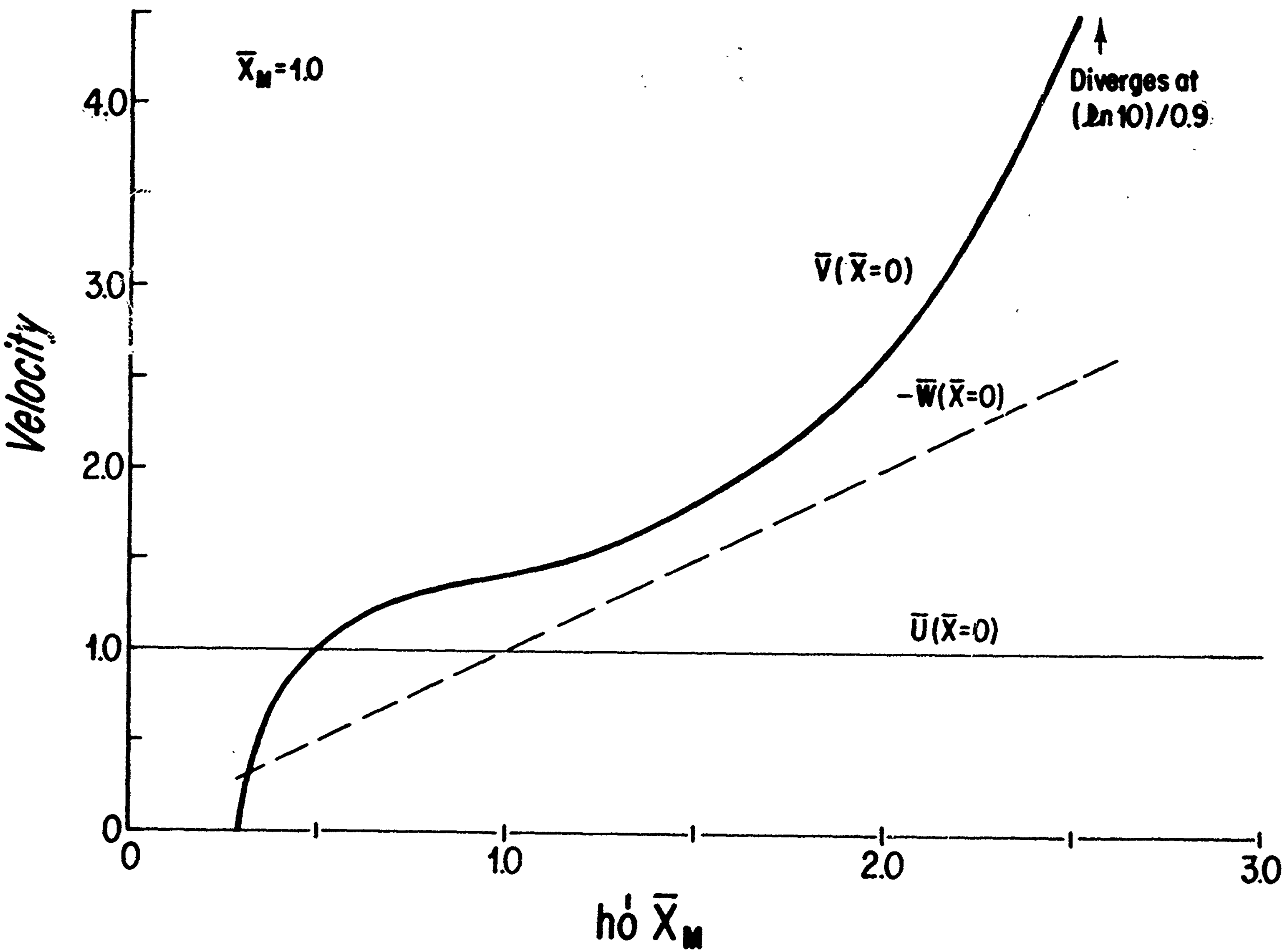
10.0
1.0
0.1
0.01
0.001
$\bar{\ell}/\bar{X}_M$
1000
100
10
Scale
0
0.60
1.20
1.80
2.40
3.00
$h_0' \bar{X}_M$



X̄/X̄_M
0
0.20
0.40
0.60
0.80
1.00
1.20
h̄
0
0.5
1.0
0.001
0.01
0.40
0.499
0.80
1.01
3.0
hδ X̄_M=2.556
h_M=200m
X_M=200Km



$\bar{X} / \bar{X}_M$
0.0
0.1
0.2
0.3
$\bar{u}(0)$
$\bar{u}(0.1)$
$\bar{u}(0.2)$
$\bar{Z}$
0.1
0.2
0.3
0.4
$h\delta \bar{X}_M = 1.01$
$h\delta = 1.01$
$\bar{X}_M = 1.0$


Velocity
X̄M=1.0
4.0
3.0
2.0
1.0
0
0
1.0
2.0
3.0
h0' X̄M
V̄(X̄=0)
-W̄(X̄=0)
Ū(X̄=0)
Diverges at
(ℓn10)/0.9